Turing Complete Neural Network based models

by **Wojciech Zaremba**

OpenAI

Need for powerful models

- Very complicated tasks require many computational steps
- Not all tasks can be solved by feed-forward network due to limited computational power

	Feed forward network	Classical CNN	CNN for detection	RNN	Neural Turing Machine
Input Size	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(n)$
Number of steps	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(n)$

More computation steps with the same number of parameters

- Reuse parameters extensively
- Few architectural choices:
    - Neural GPU;
        - Developed by Keiser et al. 2015
        - Further work by Price et al. (Summer internship at OpenAI)
    - RNN with RL (large part of my PhD)
    - Grid LSTM (Kalchbrenner et. al 2015)

Neural GPU

Neural GPU [Kaiser and Sutskever, 2015]

- The Neural GPU architecture learns arithmetic from examples.
- Feed in 6070124226526763509 0 + 4059459019222299864 3 get out 00000000000000000000101295832457490633733

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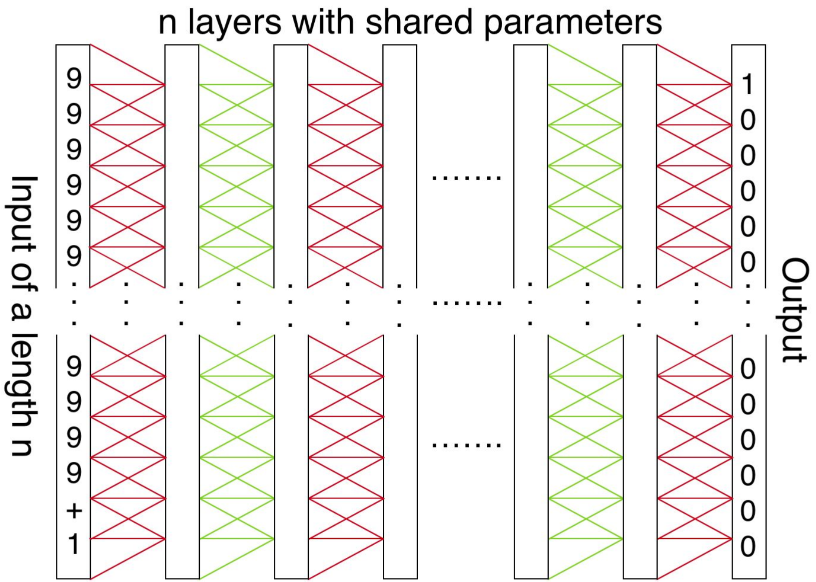
- The Neural GPU architecture learns arithmetic from examples.
- Feed in 60701242265267635090 + 40594590192222998643 get out 00000000000000000000101295832457490633733
- Can generalize to longer examples
  - Train on up to 20-digit examples
  - Still gets > 99% of 200-digit examples right.
  - (If you get lucky on training) gets > 99% of 2000-digit examples right.

Neural GPU: architecture

- Alternates between two convolutional GRUs.
- If input has size n, does 2n total convolutions. [Need at least n to pass information from one side to the other]



Neural GPU: details

- Each digit is embedded into $1 \times 4 \times F$ space, where F is the number of "filters".
    - Input becomes $n \times 4 \times F$; convolution is 2D over the $n \times 4$.

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- Dropout.

Neural GPU: Known Results

Problem	Base	24 filters	128 filters
Addition	2	Struggles	Works
	10	Fails	Works
Multiplication	2	Struggles	Works
	10	Fails	Fails

- Can we learn harder tasks?
  - What can we learn with bigger models?
  - What can we learn with smarter training?

Bigger models

- NeuralGPU barely fits into memory

- Bigger models require storing intermediate activations on CPU (tf.while_loop with swap memory options)

- Difficult to determine success due to huge non-determinism
  - Run large pool of experiments (once, we almost spent $0.5mln on them)

Bigger models

Problem	Base	24	128	256	512
Multiplication	2	Struggles	Works		
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Bigger models

Problem	Base	24	128	256	512
Multiplication	2	Struggles	Works	Works	Works
	10	Fails	Fails	Fails	Struggles?

How to do smarter training ?

- Extensive Curriculum
  - Curriculum through length (people used to do it)
  - Transfer from addition to multiplication doesn’t work
  - Transfer from small base to large seems to work

Bigger models and curriculum

Curriculum	128	256	512
10	Fails	Fails	Struggles?

Bigger models and curriculum

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10	Fails	Fails	Struggles?
$2 \rightarrow 10$	Fails	Struggles	Struggles?

Bigger models and curriculum

Curriculum	128	256	512
10	Fails	Fails	Struggles?
$2 \to 10$	Fails	Struggles	Struggles?
$2 \to 5 \to 10$	Struggles	Works	Works?

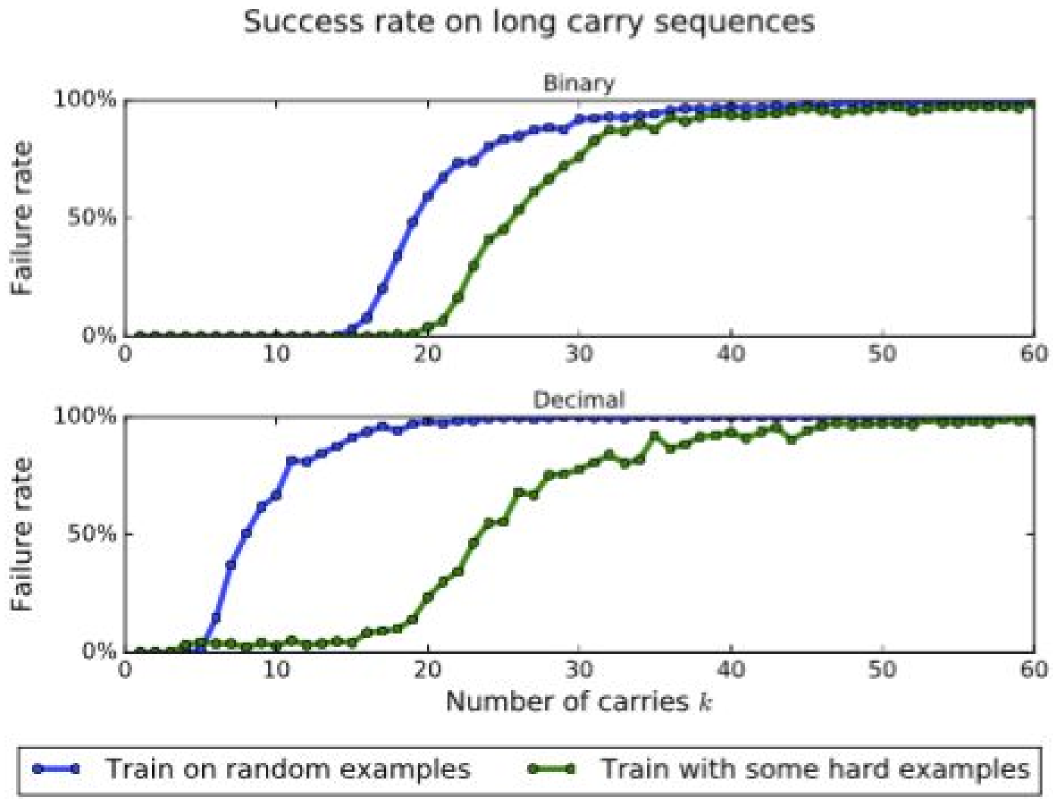
Bigger models and curriculum

Curriculum	128	256	512
10	Fails	Fails	Struggles?
$2 \to 10$	Fails	Struggles	Struggles?
$2 \to 5 \to 10$	Struggles	Works	Works?
$2 \to 4 \to 10$	Works	Works	Works?

Issues with neural GPU

- Trained on random inputs, it works reliably only on random inputs.
  - When doing addition, it cannot carry many bits.
  - Has issues with long stretches of similar digits.

Issues with carries



Issues with long similar stretches

- What is 59353073470806611971398236195285989083458222209939343360871730649133714199298764 × 71493004928584356509100241005385920385829595055047086568280792309308597157524754?

Issues with long similar stretches

- What is 593530734708066119713982361952859890834582222099393433608717306491337141992987 64 × 714930049285843565091002410053859203858295950550470865682807923093085971575247 54?
    - 424332957417500652862392857230327112302355162 72….1254256915245098421571902495277160 4056

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- What is 2×1?

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- What is 2×1?
  - 002

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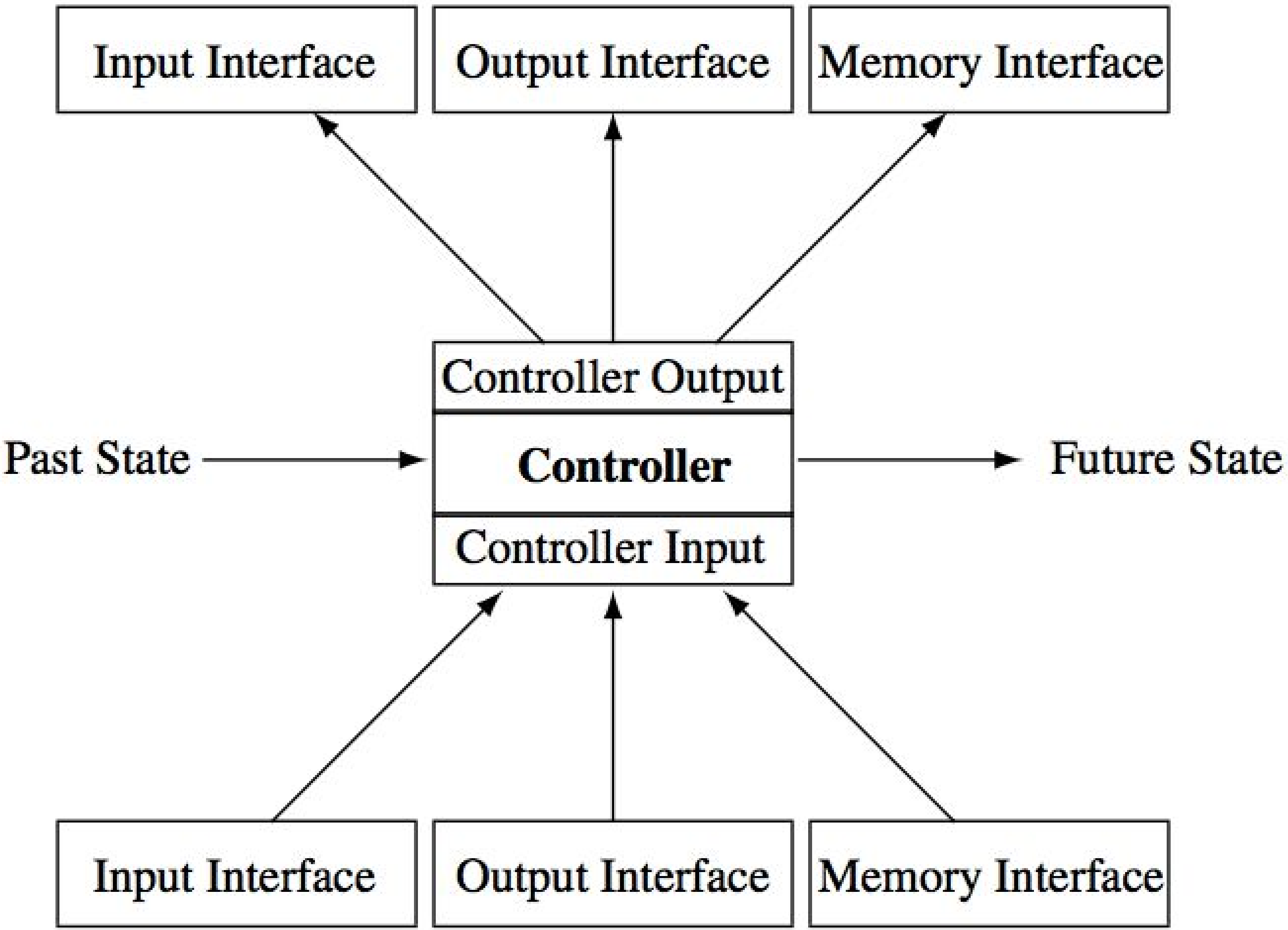
    - 0…..0017666666668850…..014

RNN with RL



Input Interface
Output Interface
Memory Interface
Controller Output
Past State
Controller
Future State
Controller Input
Input Interface
Output Interface
Memory Interface

Video

https://www.youtube.com/watch?v=GVe6kfJnRAw&feature=youtu.be

Q-learning

- Reward of 1 for every correct prediction, and 0 otherwise.
- Model trained with Q-learning
- Q(s, a) estimates sum of the future rewards for an action “a” in a state “s”.
- Q is the off-policy algorithm (remarkable)

$$Q_{t+1}(s,a) = Q_t(s,a) - \alpha\big[Q_t(s,a) - \big(R(s') + \gamma \max_a Q_n(s',a)\big)\big]$$

Q-learning as off-policy

- Policy induced by Q is the argmax_a Q(s, a)
- When we follow induced policy, we say that we are on-policy
- When we follow a different policy, we say that we are off-policy
- Q converges to Q for the optimal policy regardless of policy that we follow (as long as we can visit every state-action pair) !!!

Watkins Q(lambda)[11]

- Typical policy is a combination of on-policy (95%) with a random uniform policy (5%).
- Most of the time, we are on-policy
- This allows to regress Q on the other estimate:

$$Q^*(s_t, a_t) = \sum_{i=1}^{T} \gamma^{i-1} R(s_{t+i}) + \gamma^T \max_a Q^*(s_{t+n+1}, a)$$

[11] “Reinforcement learning: An introduction” Sutton and Barto

Dynamic Discount

- In Q-learning, the model has to predict the sum of future rewards.
- However, the length of the episode might vary.
- We reparametrize Q, so it estimates the sum of future rewards divided by number of predictions left: $$\hat{Q}(s,a) := \frac{Q(s,a)}{\hat{V}(s)}$$

Curriculum[4]

- Three row addition was unsolvable in the original form
- We start with small numbers that do not require carry.

$$\boxed{\begin{matrix}1\\2\\2\end{matrix}}\;;\;\boxed{\begin{matrix}2\\0\\2\end{matrix}}\;;\;\boxed{\begin{matrix}8&3\\3&3\\3&7\end{matrix}}\;;\;\boxed{\begin{matrix}3&2&0&6&9\\1&3&1&3&1\\2&8&0&8&3\end{matrix}}\;;\;\boxed{\begin{matrix}8&0&1&8&5&2&0&2&1\\1&3&1&4&0&7&0&5&4\\3&1&3&2&7&5&0&7&1\end{matrix}}$$

[4] "Curriculum learning.", Bengio et al.

Task											
	Test length	100	100	100	100	100	100	100	100	1000	1000
	#Units	600	400	200	200	200	200	200	200	200	200
	Discount γ	1	1	1	0.99	0.95	D	D	D	D	D
	Watkins $Q(\lambda)$	✗	✗	✗	✗	✗	✗	✓	✓	✓	✓
Task	Penalty	✗	✗	✗	✗	✗	✗	✗	✓	✗	✓
Copying		30%	60%	90%	50%	70%	90%	100%	100%	100%	100%
Reverse		0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Reverse (FF controller)		0%	0%	0%	0%	0%	0%	100%	90%	100%	90%
Walk		0%	0%	0%	0%	0%	0%	10%	90%	10%	80%
Walk (FF controller)		0%	0%	0%	0%	0%	0%	100%	100%	100%	100%
2-row Addition		10%	70%	70%	70%	80%	60%	60%	100%	40%	100%
3-row Addition		0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
3-row Addition (extra curriculum)		0%	50%	80%	40%	50%	50%	80%	80%	10%	60%
Single Digit Multiplication		0%	0%	0%	0%	0%	100%	100%	100%	0%	0%

Reinforce[12]

Objective of Reinforce:

$$\sum_{a_1,\ldots,a_n} p(a_1, \ldots, a_n|\theta) \sum_i r_i$$

we access it through sampling:

$$\mathbb{E}_a \sum_i r_i$$

[12] “Simple statistical gradient-following algorithms for connectionist reinforcement learning”, Williams

Reinforce

Derivative:

$$\sum_a p'(a|\theta) \sum_i r_i + \sum_a p(a|\theta) \sum_i r_i'$$

$$p' = p(\log p)'$$

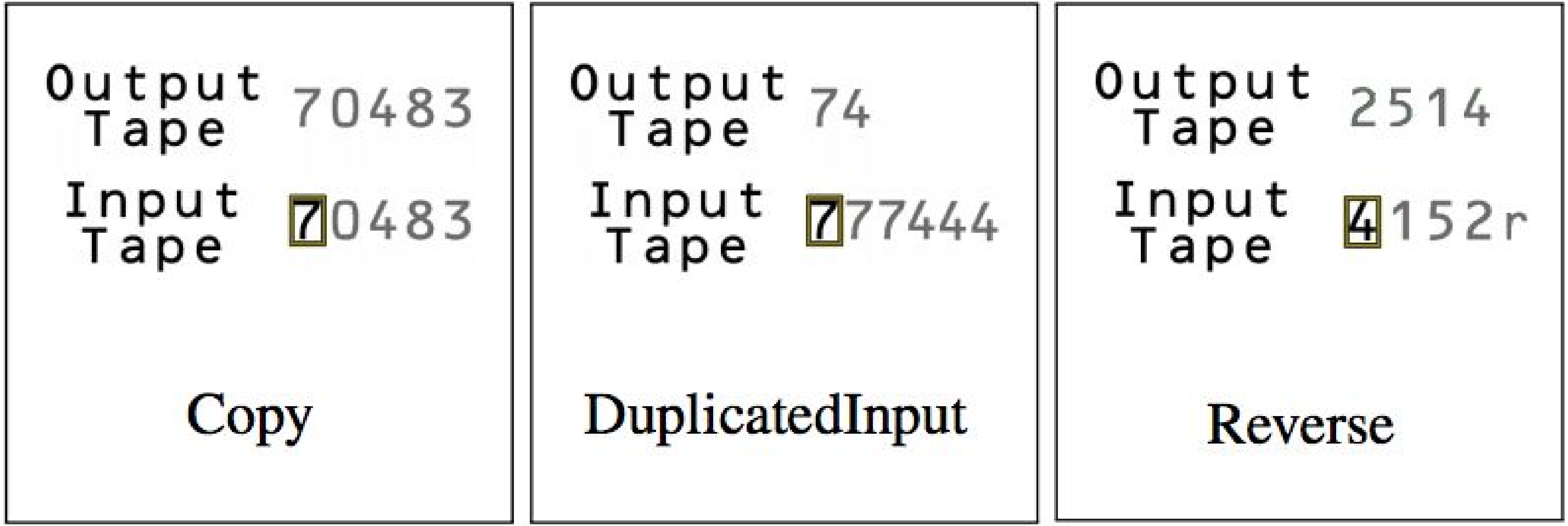
we access it through sampling:

$$\mathbb{E}_a \log p' \sum_i r_i + \sum_i r_i'$$

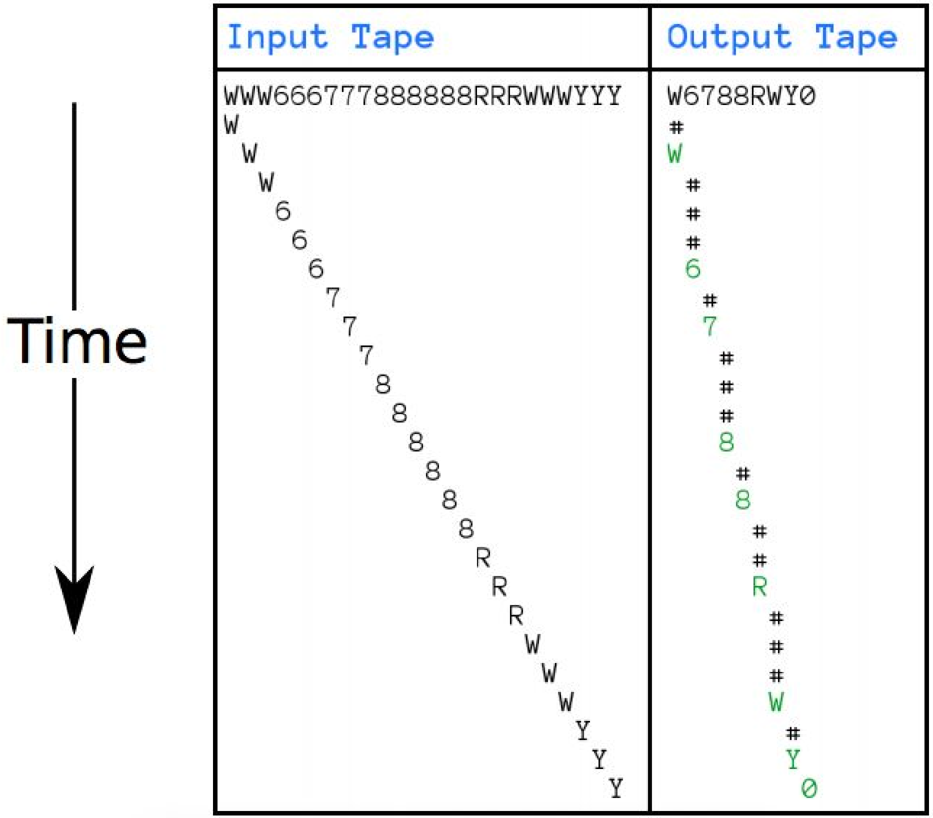
Training

- Trained with SGD
- Curriculum learning is critical
- Not easy to train (due to variance coming from sampling)
  - Various techniques to decrease variance[13]

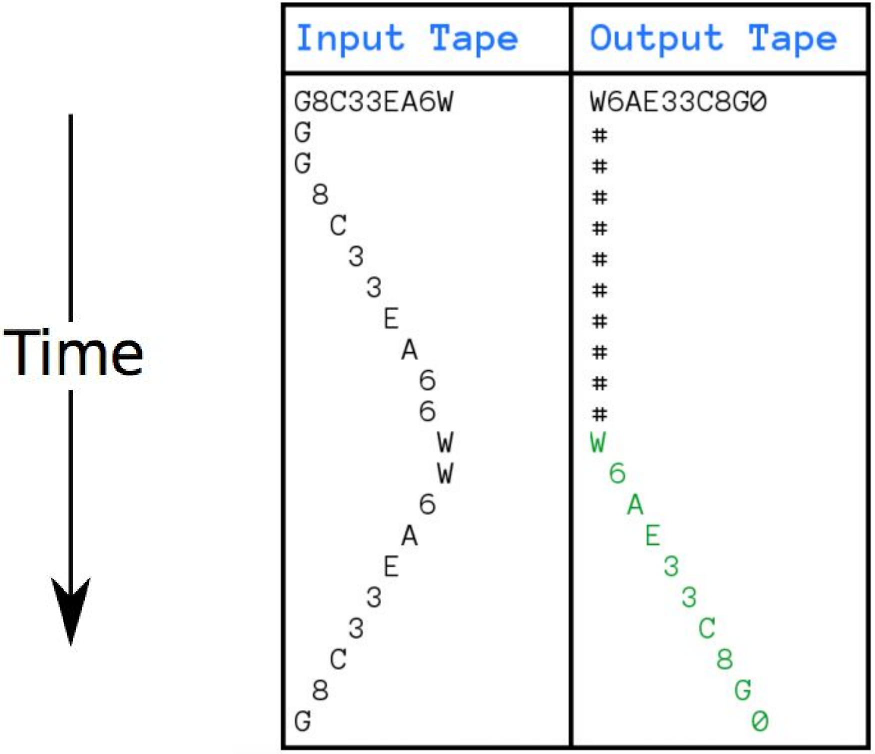
[13] “Policy Gradient Methods for Robotics” Peters and Schaal


Output Tape
70483
Input Tape
70483
Copy
Output Tape
74
Input Tape
777444
DuplicatedInput
Output Tape
2514
Input Tape
4152r
Reverse

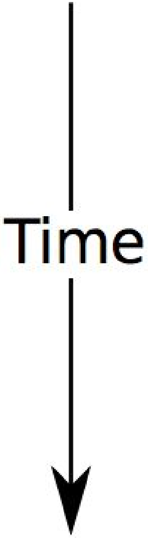
Task - DuplicatedInput

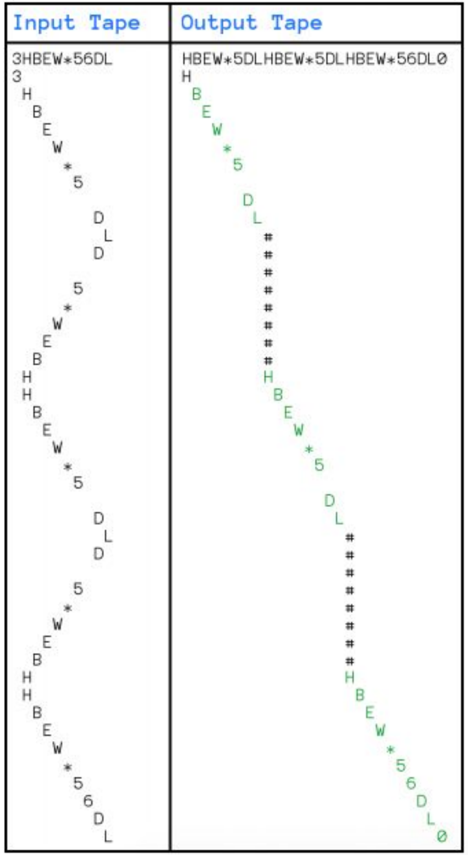


Task - Reverse



Task - RepeatCopy

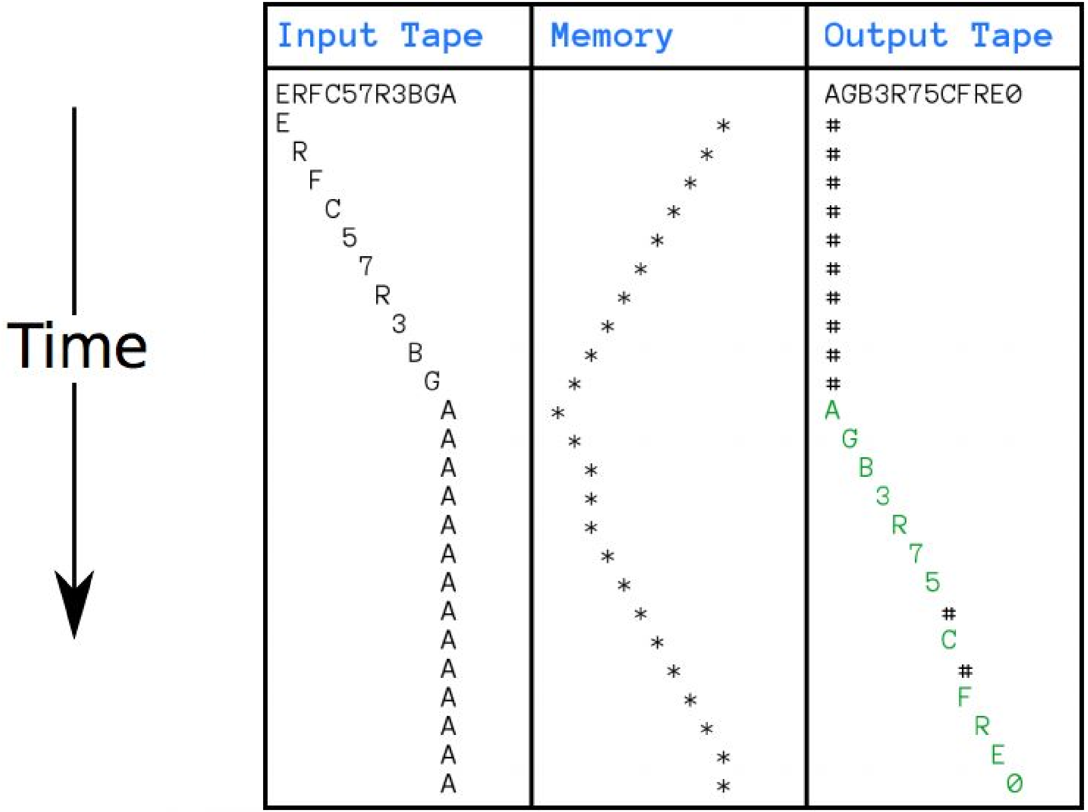




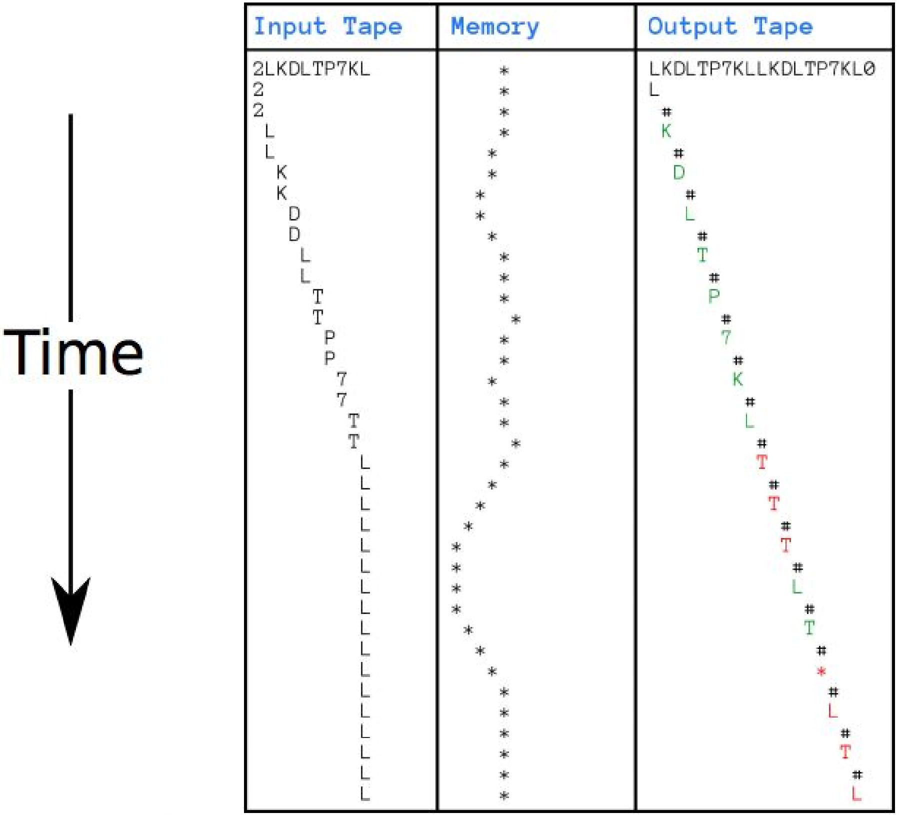
Memory interface

- Memory is a tape with 3 actions, go to the left, stay, go to the right
- Controller always reads from the previous memory location, and always saves to the next memory location
- It stores a high dimensional vector through which we backpropagate

Task - Reverse with memory



Task. RepeatCopy with memory. Failure



Gradient Checking - motivation

- Very simple to make a mistake in the implementation

- How to verify a stochastic algorithm?

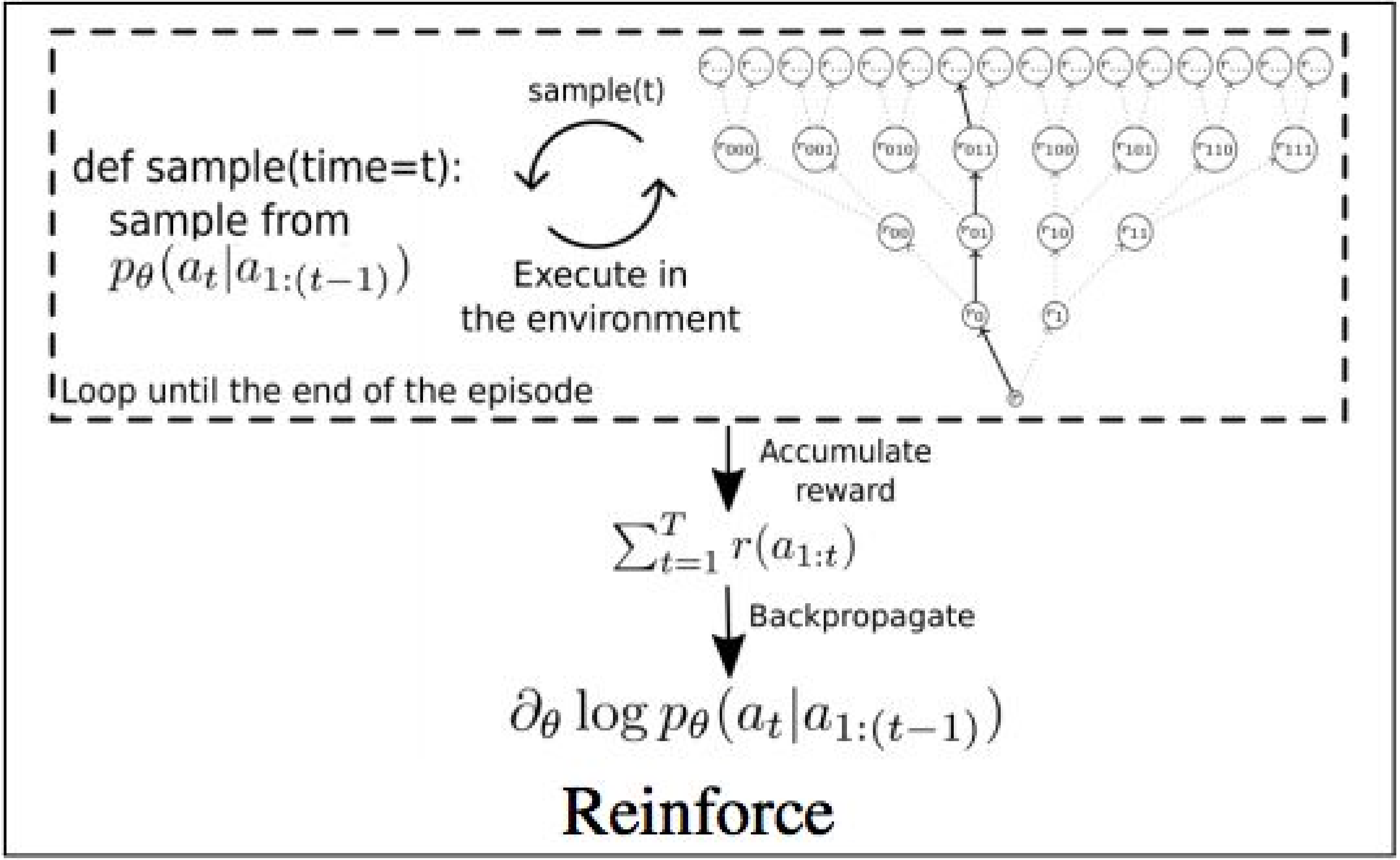
Gradient Checking for Reinforce

- We could sample actions many times and compare the average gradient to average of the numerical gradient.

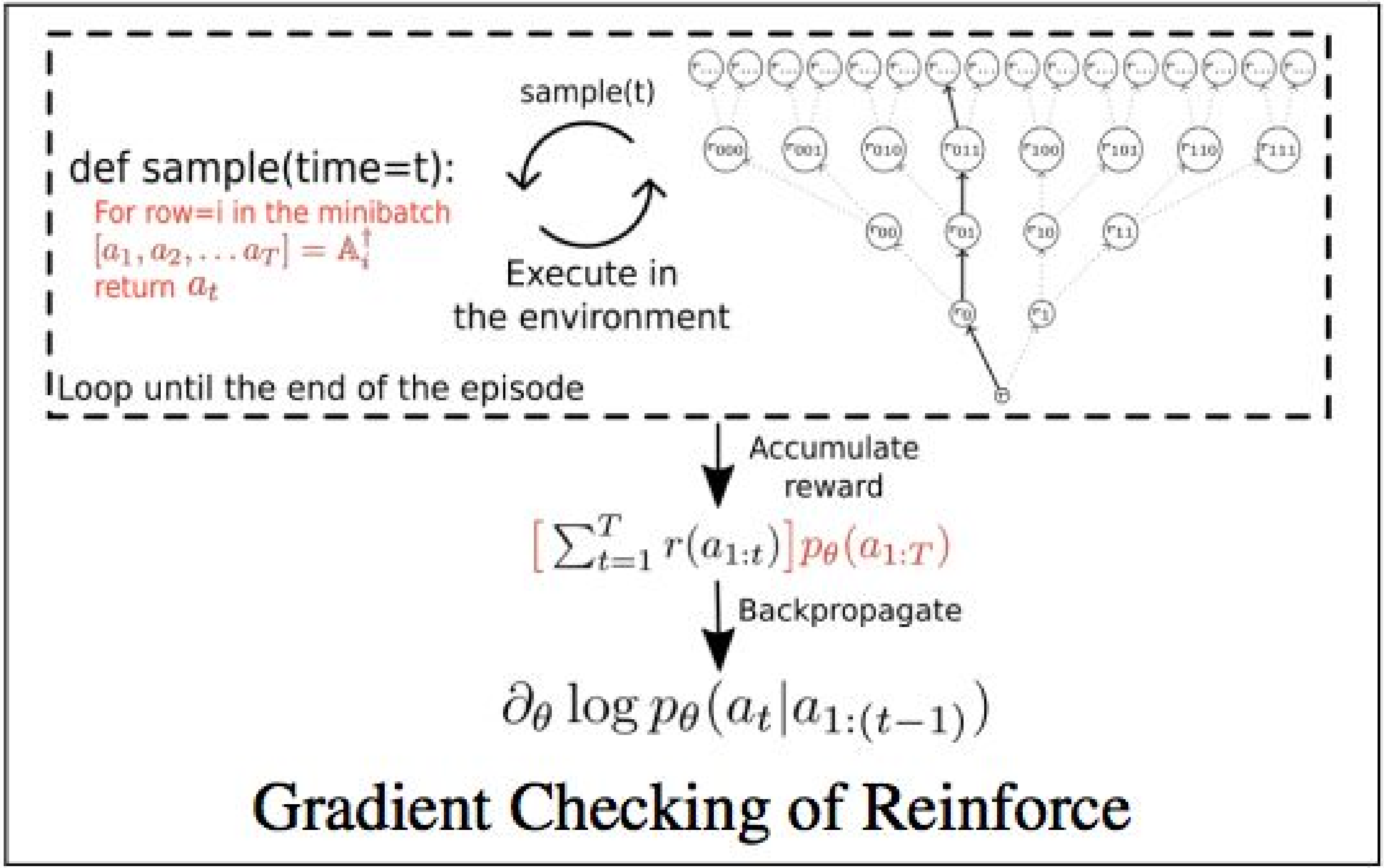
Gradient Checking for Reinforce

- We could sample actions many times and compare the average gradient to average of the numerical gradient.

- Impractical. To get good precision we would need millions of samples.



def sample(time=t):
sample from
$p_\theta(a_t|a_{1:(t-1)})$
sample(t)
Execute in
the environment
Loop until the end of the episode
Accumulate
reward
$\sum_{t=1}^{T} r(a_{1:t})$
Backpropagate
$\partial_\theta \log p_\theta(a_t|a_{1:(t-1)})$
Reinforce



def sample(time=t):
For row=i in the minibatch
$[a_1, a_2, \ldots a_T] = \mathbb{A}_i^{\dagger}$
return a_t
sample(t)
Execute in the environment
Loop until the end of the episode
Accumulate reward
$\left[\sum_{t=1}^{T} r(a_{1:t})\right] p_\theta(a_{1:T})$
Backpropagate
$\partial_\theta \log p_\theta(a_t | a_{1:(t-1)})$
Gradient Checking of Reinforce

Gradient Checking for Reinforce

- It was critical to make the model work.
- We can limit the size of the action space during gradient checking
- Gradient checking takes seconds

Q&A

- NeuralGPU
- Bigger -> better
- Curriculum
- Adversarial examples for NeuralGPU
- Q-learning
  - Dynamic discount
  - Watkins Q(lambda)
- Reinforce
- Memory
- Gradient checking

Thanks to Eric Price, Ilya Sutskever and Rob Fergus